REAL NUMBERS

EXERCISE 1.1

QUESTION 1. *Use Euclid's division algorithm to find the H.C.F. of:*

(i) 135 and 225 *(ii) 196 and 38220* *(iii) 867 and 225*

SOLUTION. **(*i*)** We start with the larger number 225.

By Euclid's Division Algorithm, we have

$$225 = 135 \times 1 + 90$$

We apply Euclid's Division Algorithm on Divisor 135 and the remainder 90.

$$135 = 90 \times 1 + 45$$

Again we apply Euclid's Division Algorithm on Divisor 90 and remainder 45

$$90 = 45 \times 2 + 0$$

H.C.F. (225, 90) = **45**

So, H.C.F of 225 and 135 is 45.

135) 225 (1 — 135 — 90

90) 135 (1 — 90 — 45

45) 90 (2 — 90 — 0

(*ii*) We have :

Division = 38220 and Divisor = 196

$$38220 = 196 \times 195 + 0$$

Hence, H.C.F. (196, 38220) = **196**

196) 38220 (195 — 196 — 1862 — 1764 — 980 — 980 — 0

(*iii*) By Euclid's Division Algorithm, we have

$$867 = 255 \times 3 + 102$$

We apply Euclid's Division Algorithm on the Divisor 255 and the remainder 102.

$$255 = 102 \times 2 + 51$$

Again we apply Euclid's Division Algorithm on the Divisor 102 and the remainder 51.

$$102 = 51 \times 2 + 0$$

H.C.F. (867, 255) = H.C.F. (255, 102) = H.C.F. (102, 51) = **51.**

255) 867 (3 — 765 — 102

102) 255 (2 — 204 — 51

51) 102 (2 — 102 — 0

QUESTION 2. *Show that any positive odd integer is of the form 6q + 1 or 6q + 3 or 6q + 5, where q is some integer.*

SOLUTION. By Euclid division Algorithm, we have

$$a = bq + r \qquad \ldots (1),\ 0 \le r < b$$

On putting, $b = 6$ in (1), we get

$$a = 6q + r \qquad [0 \le r < 6 \text{ i.e., } r = 0, 1, 2, 3, 4, 5]$$

If $r = 0$, $a = 6q$, $6q$ is divisible by 6 $\Rightarrow$ $6q$ is even.

If $r = 1$, $a = 6q + 1$, $6q + 1$ is not divisible by 2.

If $r = 2$, $a = 6q + 2$, $6q + 2$ is divisible by 2 $\Rightarrow$ $6q + 2$ is even.

If $r = 3$, $a = 6q + 3$, $6q + 3$ is not divisible by 2.

If $r = 4$, $a = 6q + 4$, $6q + 4$ is divisible by 2 $\Rightarrow$ $6q + 4$ is even.

If $r = 5$, $a = 6q + 5$, $6q + 5$ is not divisible by 2.

Since, $6q$, $6q + 2$, $6q + 4$ are even.

Hence, **the remaining integers $6q + 1$, $6q + 3$, $6q + 5$ are odd.** **Proved.**

QUESTION 3. ***An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march ?***

SOLUTION. To find the maximum number of columns, we have to find the H.C.F. of 616 and 32.

$$\begin{array}{r|l|l} 32 & 616 & 19 \\ & \underline{32} & \\ & 296 & \\ & \underline{288} & \\ & \underline{8} & \end{array} \qquad \begin{array}{r|l|l} 8 & 32 & 4 \\ & \underline{32} & \\ & \underline{0} & \end{array}$$

i.e., $616 = 32 \times 19 + 8$ *i.e.*, $32 = 8 \times 4 + 0$

$\therefore$ The H.C.F. of 616 and 32 is 8.

Hence, maximum number of columns is **8.** **Ans.**

QUESTION 4. ***Use Euclid's Division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.***

SOLUTION. By Euclid Division Algorithm, we have

$$a = bq + r \qquad \text{... (1)}$$

On putting $b = 3$ in (1), we get

$$a = 3q + r, [0 \le r < 3, \ i.e., r = 0, 1, 2]$$

If $r = 0$ $a = 3q$ $\Rightarrow$ $a^2 = 9q^2$ ... (2)

If $r = 1$ $a = 3q + 1$ $\Rightarrow$ $a^2 = 9q^2 + 6q + 1$ ... (3)

If $r = 2$ $a = 3q + 2$ $\Rightarrow$ $a^2 = 9q^2 + 12q + 4$ ... (4)

From (2), $9q^2$ is a square of the form $3m$, where $m = 3q^2$

From (3), $9q^2 + 6q + 1$ *i.e.*, $3(3q^2 + 2q) + 1$ is a square which is of the form $3m + 1$ where $m = 3q^2 + 2q$

From (4), $9q^2 + 12q + 4$ *i.e.*, $3(3q^2 + 2q + 1) + 1$ is a square which is of the form $3m + 1$, where $m = 3q^2 + 4q + 1$.

$\therefore$ **The square of any +ve integer is either of the form 3*m* or 3*m* + 1 for some integer *m*.** **Proved.**

QUESTION 5. ***Use Euclid's Division lemma to show that the cube of any integer is either of the form 9m, 9m + 1 or 9m + 8.***

SOLUTION. Let m be any positive integer. Then it is of the form $3m$, $3m + 1$ or $3m + 2$. Now, we have to prove that the cube of each of these can be rewritten in the form $9q$, $9q + 1$ or $9q + 8$.

Now, $(3m)^3 = 27m^3 = 9(3m^3)$
$= 9q$, where $q = 3m^3$

$(3m + 1)^3 = (3m)^3 + 3(3m)^2 \cdot 1 + 3(3m) \cdot 1^2 + 1$
$= 27m^3 + 27m^2 + 9m + 1$
$= 9(3m^3 + 3m^2 + m) + 1$
$= 9q + 1$, where $q = 3m^3 + 3m^2 + m$

and $(3m + 2)^3 = (3m)^3 + 3(3m)^2 \cdot 2 + 3(3m) \cdot 2^2 + 8$
$= 27m^3 + 54m^2 + 36m + 8$
$= 9(3m^3 + 6m^2 + 4m) + 8$
$=$ **$9q + 8$,** where $q = 3m^3 + 6m^2 + 4m$ **Ans.**

EXERCISE 1.2

QUESTION 1. *Express each number as a product of its prime factors :*

(i) 140 *(ii) 156* *(iii) 3825* *(iv) 5005* *(v) 7429*

SOLUTION. (*i*) We use the division method as shown below :

2	140
2	70
5	35
7	7
	1

$\therefore$ 140 = 2 × 2 × 5 × 7

= $\mathbf{2^2 \times 5 \times 7}$

(*ii*) We use the division method as shown below :

2	156
2	78
3	39
13	13
	1

$\therefore$ 156 = 2 × 2 × 3 × 13

= $\mathbf{2^2 \times 3 \times 13}$

(*iii*) We use the division method as shown below :

3	3825
3	1275
5	425
5	85
17	17
	1

$\therefore$ 3825 = 3 × 3 × 5 × 5 × 17

= $\mathbf{3^2 \times 5^2 \times 17}$

(*iv*) We use the division method as shown below :

5	5005
7	1001
11	143
13	13
	1

$\therefore$ 5005 = **5 × 7 × 11 × 13**

(*v*) We use the division method as shown below :

17	7429
19	437
23	23
	1

$\therefore$ 7429 = **17 × 19 × 23** **Ans.**

QUESTION 2. *Find the L.C.M. and H.C.F. of the following pairs of integers and verify :*

L.C.M. × H.C.F. = Product of the two numbers.

(i) 26 and 91 *(ii) 510 and 92* *(iii) 336 and 54*

SOLUTION. (*i*) 26 and 91

2	26
13	13
	1

7	91
13	13
	1

26 = 2 × 13 and 91 = 7 × 13

$\therefore$ L.C.M. of 26 and 91 = 2 × 7 × 13 = 182

and H.C.F. of 26 and 91 = 13

Now, 182 × 13 = 2366 and 26 × 91 = 2366

Hence, **182 × 13 = 26 × 91** **Proved.**

(*ii*) 510 and 92

2	510
3	255
5	85
17	17
	1

2	92
2	46
23	23
	1

510 = 2 × 3 × 5 × 17 and 92 = 2 × 2 × 23

$\therefore$ L.C.M. of 510 and 92 = 2 × 2 × 3 × 5 × 17 × 23 = **23460**

and H.C.F. of 510 and 92 = **2**

Now, 23460 × 2 = 46920 and 510 × 92 = 46920

Hence, **23460 × 2 = 510 × 92** **Proved.**

(*iii*) 336 and 54

2	336
2	168
2	84
2	42
3	21
7	7
	1

2	54
3	27
3	9
3	3
	1

336 = 2 × 2 × 2 × 2 × 3 × 7

and 54 = 2 × 3 × 3 × 3

$\therefore$ L.C.M. of **336** and **54** = 2 × 2 × 2 × 2 × 3 × 3 × 3 ×7

= **3024**

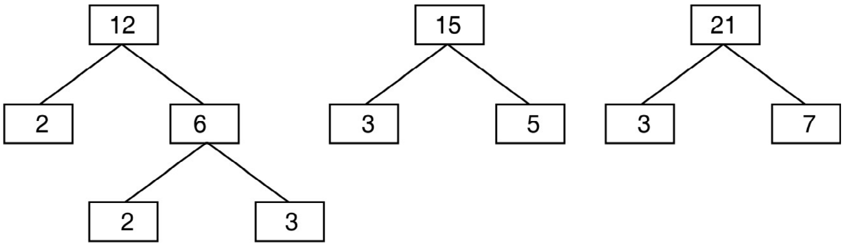
and H.C.F. of 336 and 54 = 2 × 3 = **6**

Now, 3024 × 6 = 18144 and 336 × 54 = 18144

Hence, **3024 × 6 = 336 × 54** **Proved.**

QUESTION 3. ***Find the H.C.F. and L.C.M. of the following integers by applying prime factorisation method.***
(i) 12, 15, 21; (ii) 17, 23, 29 (iii) 8, 9 and 25

SOLUTION. (*i*)



$12 = 2 \times 2 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$

Here 3 is common prime factor of the given numbers.
Hence, H.C.F. = 3
H.C.F. (12, 15, 21) = 3
L.C.M. is product of the prime factors $2 \times 2 \times 3 \times 7 \times 5$
The common factor 3 is repeated three times, but 3 in multiplication is written once.
L.C.M. (12, 15, 21) = 420

(*ii*) 17 23 29
H.C.F. There is no common factor as 17, 23, 29 they are primes. Hence H.C.F. is 1.
L.C.M. is the product of all prime factors 17, 23 and 29.
$17 \times 23 \times 29 = 11339$
Hence, **H.C.F. (17, 23, 29) = 1,**
L.C.M (17, 23, 29) = 11339.

(*iii*) First we write the prime factorisation of each of the given numbers.
$8 = 2 \times 2 \times 2 = 2^3$, $9 = 3 \times 3 = 3^2$, $25 = 5 \times 5 = 5^2$
$\therefore$ L.C.M. = $\mathbf{2^3 \times 3^2 \times 5^2 = 8 \times 9 \times 25 = 1800}$
and **H.C.F. = 1.** **Ans.**

QUESTION 4. ***Given H.C.F. (306, 657) = 9, find L.C.M. (306, 657).***

SOLUTION. We have, H.C.F. (306, 657) = 9.
We know that,
Product of L.C.M. and H.C.F. = Product of two numbers.
$\Rightarrow$ L.C.M. $\times 9 = 306 \times 657$
$$\Rightarrow \quad \text{L.C.M.} = \frac{306 \times 657}{9} = 22338$$
Hence, **L.C.M. (306, 657) = 22338.** **Ans.**

QUESTION 5. ***Check whether 6^n can end with the digit 0 or any $n \in N$.***

SOLUTION. If the number 6^n ends with the digit zero. Then it is divisible by 5. Therefore the prime factorisation of 6^n contains the prime 5. This is not possible because the only primes in the factorisation of 6^n are 2 and 3 and the uniqueness of the fundamental theorem of arithmetic guarantees that there are no other prime in the factorisation of 6^n.
So, there is no value of n in natural numbers for which 6^n ends with the digit zero. **Ans.**

QUESTION 6. ***Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.***

SOLUTION. We have, $7 \times 11 \times 13 + 13 = 1001 + 13 = 1014$

$$1014 = 2 \times 3 \times 13 \times 13$$

So, it is the product of more than two prime numbers.

2, 3 and 13.

Hence, **it is a composite number.**

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5040 + 5 = 5045$

$\Rightarrow \quad 5045 = 5 \times 1009$

It is the product of prime factors 5 and 1009.

Hence, **it is a composite number.** **Ans.**

QUESTION 7. ***There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point ?***

SOLUTION. They will be again at the starting point at least common multiples of 18 and 12 minutes. To find the LCM of 18 and 12, we have:

2	18
3	9
3	3
	1

2	12
2	6
3	3
	1

$18 = 2 \times 3 \times 3$ and $12 = 2 \times 2 \times 3$

L.C.M. of 18 and 12 $= 2 \times 2 \times 3 \times 3 = 36$

So, Sonia and Ravi will meet again at the starting point after **36 minutes.** **Ans.**

EXERCISE 1.3

QUESTION 1. ***Prove that $\sqrt{5}$ is an irrational number by contradiction method.***

SOLUTION. Suppose $\sqrt{5}$ represents a rational number. Then $\sqrt{5}$ can be expressed in the form $\frac{p}{q}$, where p, q are integers and have no common factor, $q \neq 0$.

$$\sqrt{5} = \frac{p}{q}$$

Squaring both sides, we get

$$5 = \frac{p^2}{q^2} \quad \Rightarrow \quad p^2 = 5q^2 \quad \text{... (1)}$$

$\Rightarrow \quad$ 5 divides p^2

$\Rightarrow \quad$ 5 divides p. ... (2) (*Concept 3*)

Let $p = 5\,m \quad \Rightarrow \quad p^2 = 25\,m^2$

Putting the value of p^2 in (1), we get

$$25\,m^2 = 5\,q^2 \quad \Rightarrow \quad 5\,m^2 = q^2$$

$\Rightarrow \quad$ 5 divides $q^2 \quad \Rightarrow \quad$ 5 divides q. ... (3) (*Concept 3*)

Thus from (2), 5 divides p and from (3), 5 also divides q. It means 5 is a common factor of p and q. This contradicts the supposition so there is no common factor of p and q.

Hence, **$\sqrt{5}$ is an irrational number.** **Proved.**

QUESTION 2. ***Prove that $3 + 2\sqrt{5}$ is irrational.***

SOLUTION. Let us assume, to the contrary, that $3 + 2\sqrt{5}$ is a rational number.

Now, let $3 + 2\sqrt{5} = \frac{a}{b}$, where a and b are coprime and $b \neq 0$.

So, $2\sqrt{5} = \frac{a}{b} - 3$ or $\sqrt{5} = \frac{a}{2b} - \frac{3}{2}$

Since a and b are integers, therefore

$$\frac{a}{2b} - \frac{3}{2} \text{ is a rational number}$$

$\therefore$ $\sqrt{5}$ is a rational number.

But $\sqrt{5}$ is an irrational number.
This shows that our assumption is incorrect.

So, **$3 + 2\sqrt{5}$ is an irrational number.** **Proved.**

QUESTION 3. ***Prove that the following are irrationals :***

(i) $\frac{1}{\sqrt{2}}$ *(ii)* $7\sqrt{5}$ *(iii)* $6 + \sqrt{2}$

SOLUTION. **(*i*)** Let us assume, to the contrary, that $\frac{1}{\sqrt{2}}$ is rational. That is, we can find co-prime integers p and q $(q \neq 0)$ such that

$$\frac{1}{\sqrt{2}} = \frac{p}{q} \text{ or } \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{p}{q} \text{ or } \frac{\sqrt{2}}{2} = \frac{p}{q}$$

or $$\sqrt{2} = \frac{2p}{q}$$

Since p and q are integres $\frac{2p}{q}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $\frac{1}{\sqrt{2}}$ **is an irrational.** **Proved.**

(*ii*) Let us assume, to the contrary, that $7\sqrt{5}$ is rational. That is, we can find co-prime integers p and q $(q \neq 0)$ such that $7\sqrt{5} = \frac{p}{q}$.

So, $$\sqrt{5} = \frac{p}{7q}.$$

Since p and q are integers, $\frac{p}{7q}$ is rational and so is $\sqrt{5}$.

But this contradicts the fact that $\sqrt{5}$ is irrational. So, we conclude that **$7\sqrt{5}$ is an irrational.** **Proved.**

(*iii*) Let us assume, to the contrary, that $6 + \sqrt{2}$ is rational. That is, we can find integers p and q $(\neq 0)$ such that

$$6 + \sqrt{2} = \frac{p}{q} \text{ or } 6 - \frac{p}{q} = \sqrt{2}$$

or $$\sqrt{2} = 6 - \frac{p}{q}$$

Since p and q are integers, we get $6 - \frac{p}{q}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that **$6 + \sqrt{2}$ is an irrational.** **Proved.**

EXERCISE 1.4

QUESTION 1. ***Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:***

(i) $\frac{13}{3125}$ *(ii)* $\frac{17}{8}$ *(iii)* $\frac{64}{455}$ *(iv)* $\frac{15}{1600}$ *(v)* $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^2}$ *(vii)* $\frac{129}{2^2 5^7 7^5}$ *(viii)* $\frac{6}{15}$ *(ix)* $\frac{35}{50}$ *(x)* $\frac{77}{210}$

SOLUTION. *(i)* Since the factors of the denominator 3125 are $2^0 \times 5^5$. Therefore $\frac{13}{3125}$ is **a terminating decimal.**

(ii) Since the factors of the denominator 8 are $2^3 \times 5^0$. So, $\frac{17}{8}$ is a **terminating decimal.**

(iii) Since the factors of the denominator 455 is not in the form $2^n \times 5^m$. So $\frac{64}{455}$ is **non-terminating repeating decimal.**

(iv) Since the factors of the denominator 1600 are $2^6 \times 5^2$. So, $\frac{15}{1600}$ is a **terminating decimal.**

(v) Since the factors of the denominator 343 is not of the form $2^n \times 5^m$. So, it is **non-teminating repeating decimal.**

(vi) Since the denominator is of the form $2^3 \times 5^2$. So, $\frac{23}{2^3 \times 5^2}$ is a **terminating decimal.**

(vii) Since the factors of the denominator $2^2\, 5^7\, 7^5$ is not of the form $2^n \times 5^m$. So, $\frac{129}{2^2 5^7 7^5}$ is **non-terminating repeating decimal.**

(viii) $\frac{6}{15} = \frac{2}{5}$ here the factors of the denominator 5 is of the form $2^0 \times 5^1$. So, $\frac{6}{15}$ is a **terminating decimal.**

(ix) Since the factors of the denominator 50 is of the form $2^1 \times 5^2$. So, $\frac{35}{50}$ is **terminating decimal.**

(x) Since the factors of the denominator 210 is not of the form $2^n \times 5^m$. So, $\frac{77}{210}$ is **non-terminating repeating decimal.** **Ans.**

QUESTION 2. *Write down the decimal expansion of those rational numbers in Question 1 which have terminating decimal expansions.*

SOLUTION. *(i)* $\frac{13}{3125} = \frac{13}{5\times5\times5\times5\times5} = \frac{13\times2\times2\times2\times2\times2}{5\times2\times5\times2\times5\times2\times5\times2\times5\times2}$

$= \frac{13\times32}{10\times10\times10\times10\times10} = \frac{416}{100000} = \mathbf{0.0046}$

(ii) $\frac{17}{8} = \frac{17\times5^3}{2^3\times5^3} = \frac{17\times5^3}{10^3} = \frac{17\times125}{10^3} = \frac{2125}{1000} = \mathbf{2.125}$

(iii) **Non-terminating repeating.**

(iv) $\frac{15}{1600} = \frac{15}{2^6\times5^2} = \frac{15}{2^4\times2^2\times5^2} = \frac{15}{2^4\times10^2} = \frac{15\times5^4}{2^4\times5^4\times10^2} = \frac{15\times625}{10^4\times10^2} = \frac{9375}{1000000} = \mathbf{0.009375}$

(v) **Non-terminating repeating.**

(vi) $\frac{23}{2^3.5^2} = \frac{23}{2.2^2.5^2} = \frac{23}{2.10^2} = \frac{23\times5}{2\times5\times10^2} = \frac{115}{1000} = \mathbf{0.115}$

(vii) **Non-terminating repeating.**

(viii) $\frac{6}{15} = \frac{2}{5} = \frac{4}{10} = \mathbf{0.4}$ *(ix)* $\frac{35}{50} = \frac{35\times2}{50\times2} = \frac{70}{100} = \mathbf{0.70}$

(x) **Non-terminating repeating.** **Ans.**

QUESTION 3. *The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational and of the form $\frac{p}{q}$, what can you say about the prime factors of q ?*

(i) 43.123456789 *(ii) 0.120120012000120000....* *(iii)* $43.\overline{123456789}$

SOLUTION. **(*i*)** 43.123456789 **is terminating.**

So, it represents a **rational number.**

Thus, $43.123456789 = \frac{43123456789}{1000000000} = \frac{p}{q}$. Thus, $q = 10^9$.

(*ii*) 0.12012001200012000... is non-terminating and non-repeating. So, it **is an irrational.**

(*iii*) $43.\overline{12345789}$ is non-terminating but repeating. So, it **is a rational.**

Thus, $43.\overline{123456789} = \frac{4312345646}{999999999} = \frac{p}{q}$.

Thus, $q = 999999999$.